Residential Water Heaters as a Grid-Scale Energy Storage Solution Using Model Predictive Control

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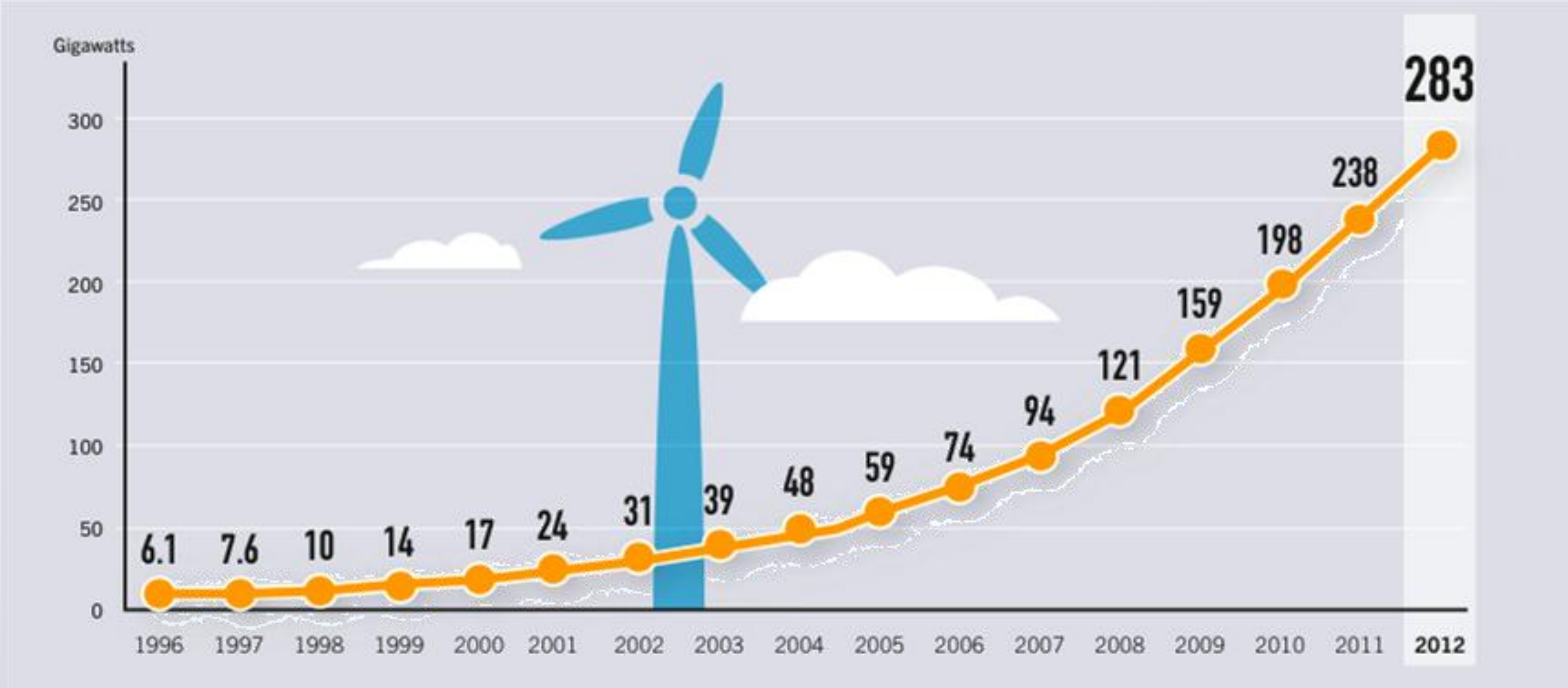
8.1.13

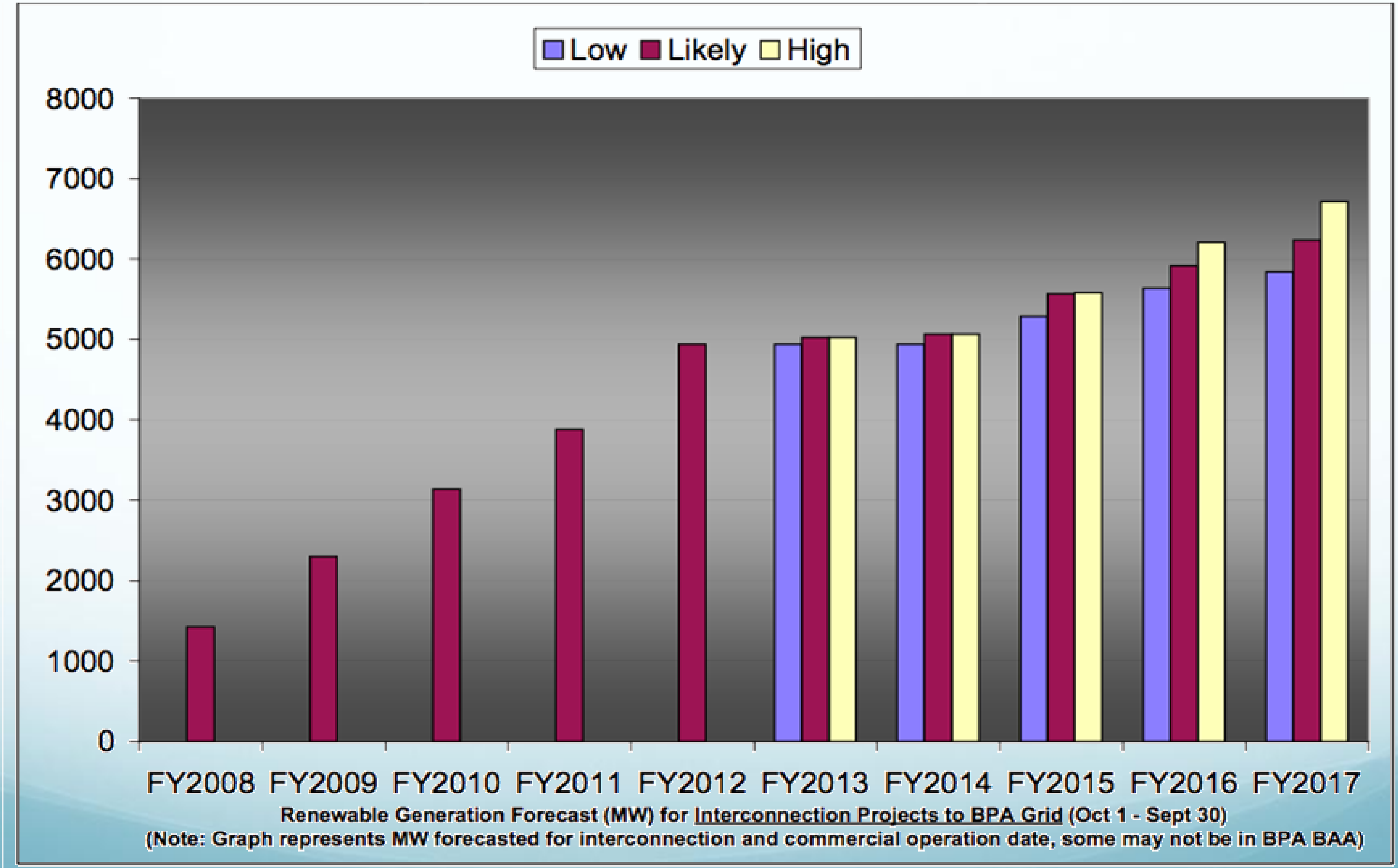
Context within the Industry

Wind Power Global Capacity



Source: REN 21 Global Status Report

Forecast of Renewable Projects Connected to BPA

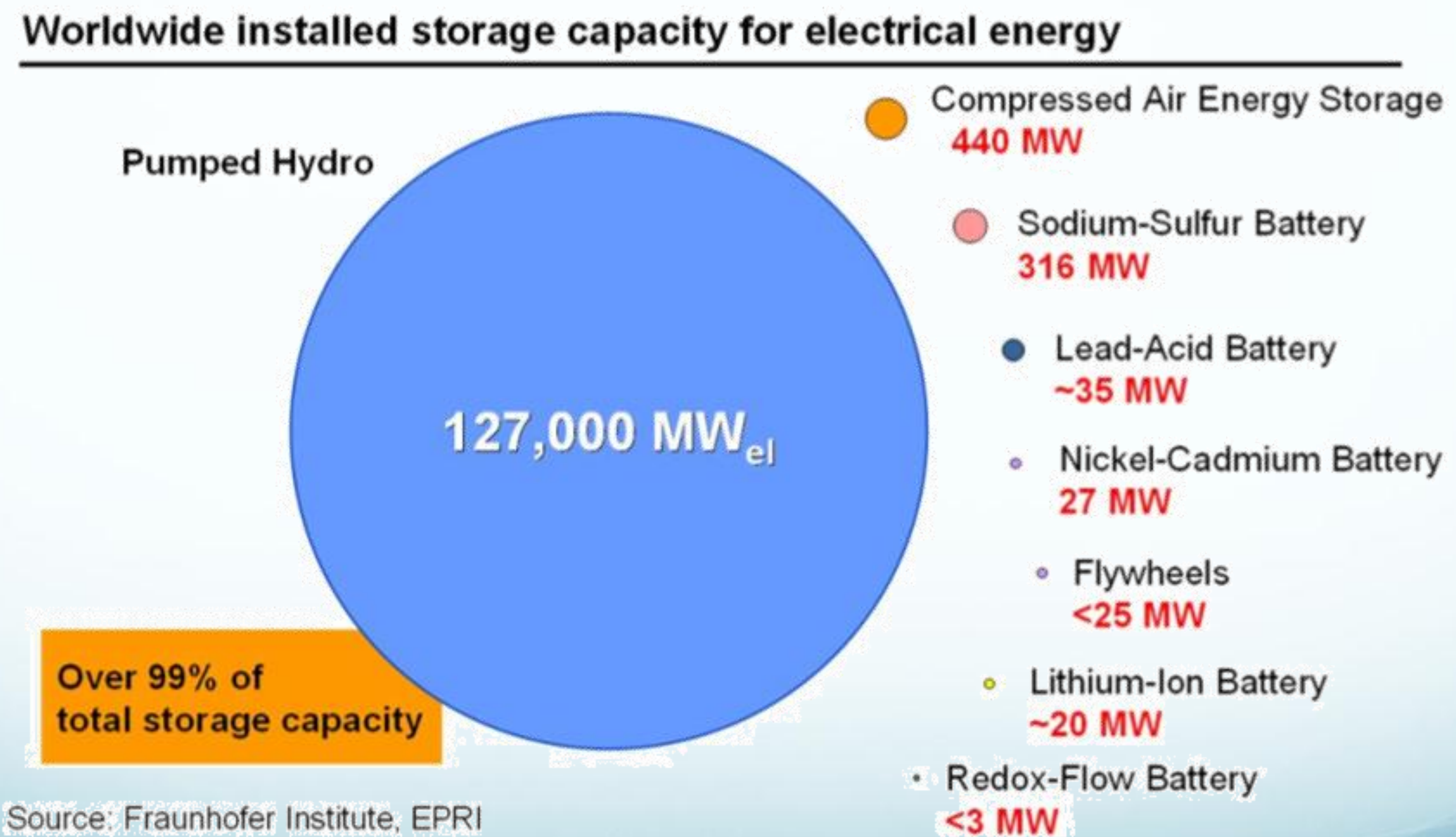


Source: Bonneville Power Administration

Reserves: FCRPS

Source: Bonneville Power Administration

Energy Storage Capacity Worldwide



Residential Water Heaters as Energy Storage

- Two 4.5 kW heating elements
  - Operated individually
- Energy storage capacity of up to 6 kWh in the lower half of the tank

Our Approach:
Model Predictive Control

Model Predictive Control

- Advantages:
  - Can use predictions to determine control actions
  - Can control multiple energy storage sources
  - Can manage many constraints and meet several objectives
- Determines control actions based on:
  - Current State
  - Predicted Behavior
  - Predicted Disturbances

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_u\mathbf{u}(k) + \mathbf{B}_v\mathbf{v}(k)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}_u\mathbf{u}(k) + \mathbf{D}_v\mathbf{v}(k)$$

MPC Formulation

$$\underbrace{\begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k+1) \\ \mathbf{y}(k+2) \\ \mathbf{y}(k+3) \\ \vdots \\ \mathbf{y}(k+H_p) \end{bmatrix}}_{\vec{\mathbf{y}}(k)} = \underbrace{\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \\ \vdots \\ \mathbf{CA}^{H_p} \end{bmatrix}}_{\mathbf{S}_x} \mathbf{x}(k)$$

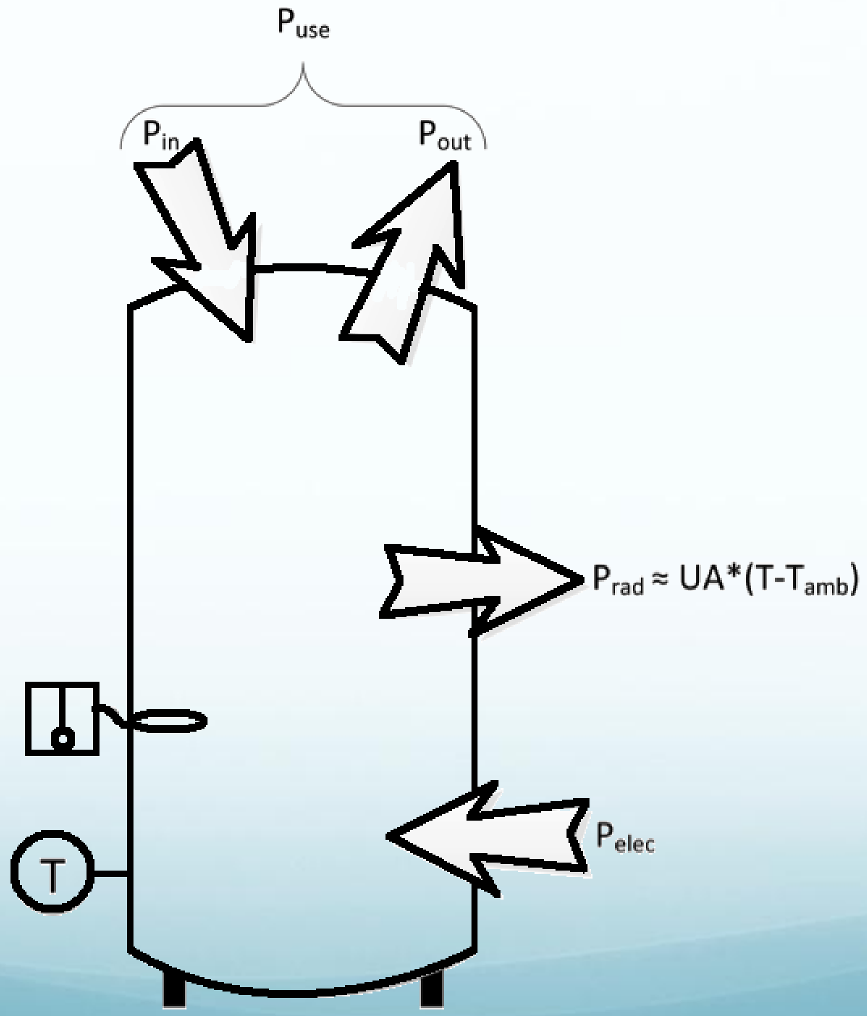
$$+ \underbrace{\begin{bmatrix} \mathbf{D}_u & 0 & 0 & 0 & \cdots & 0 \\ \mathbf{CB}_u & \mathbf{D}_u & 0 & 0 & \cdots & 0 \\ \mathbf{CAB}_u & \mathbf{CB}_u & \mathbf{D}_u & 0 & \cdots & 0 \\ \mathbf{CA}^2\mathbf{B}_u & \mathbf{CAB}_u & \mathbf{CB}_u & \mathbf{D}_u & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{H_p-1}\mathbf{B}_u & \mathbf{CA}^{H_p-2}\mathbf{B}_u & \cdots & \cdots & \mathbf{CB}_u & \mathbf{D}_u \end{bmatrix}}_{\mathbf{S}_u} \underbrace{\begin{bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \mathbf{u}(k+2) \\ \mathbf{u}(k+3) \\ \vdots \\ \mathbf{u}(k+H_p) \end{bmatrix}}_{\vec{\mathbf{u}}(k)}$$

$$+ \underbrace{\begin{bmatrix} \mathbf{D}_v & 0 & 0 & 0 & \cdots & 0 \\ \mathbf{CB}_v & \mathbf{D}_v & 0 & 0 & \cdots & 0 \\ \mathbf{CAB}_v & \mathbf{CB}_v & \mathbf{D}_v & 0 & \cdots & 0 \\ \mathbf{CA}^2\mathbf{B}_v & \mathbf{CAB}_v & \mathbf{CB}_v & \mathbf{D}_v & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{H_p-1}\mathbf{B}_v & \mathbf{CA}^{H_p-2}\mathbf{B}_v & \cdots & \cdots & \mathbf{CB}_v & \mathbf{D}_v \end{bmatrix}}_{\mathbf{S}_v} \underbrace{\begin{bmatrix} \mathbf{v}(k) \\ \mathbf{v}(k+1) \\ \mathbf{v}(k+2) \\ \mathbf{v}(k+3) \\ \vdots \\ \mathbf{v}(k+H_p) \end{bmatrix}}_{\vec{\mathbf{v}}(k)}$$

$$\vec{\mathbf{y}}(k) = \mathbf{S}_x\mathbf{x}(k) + \mathbf{S}_u\vec{\mathbf{u}}(k) + \mathbf{S}_v\vec{\mathbf{v}}(k)$$

Water Heater Model

$$V_{total} \cdot d \cdot C_P \cdot \frac{dT}{dt} = \eta P_{elec} - P_{use} - P_{rad}$$

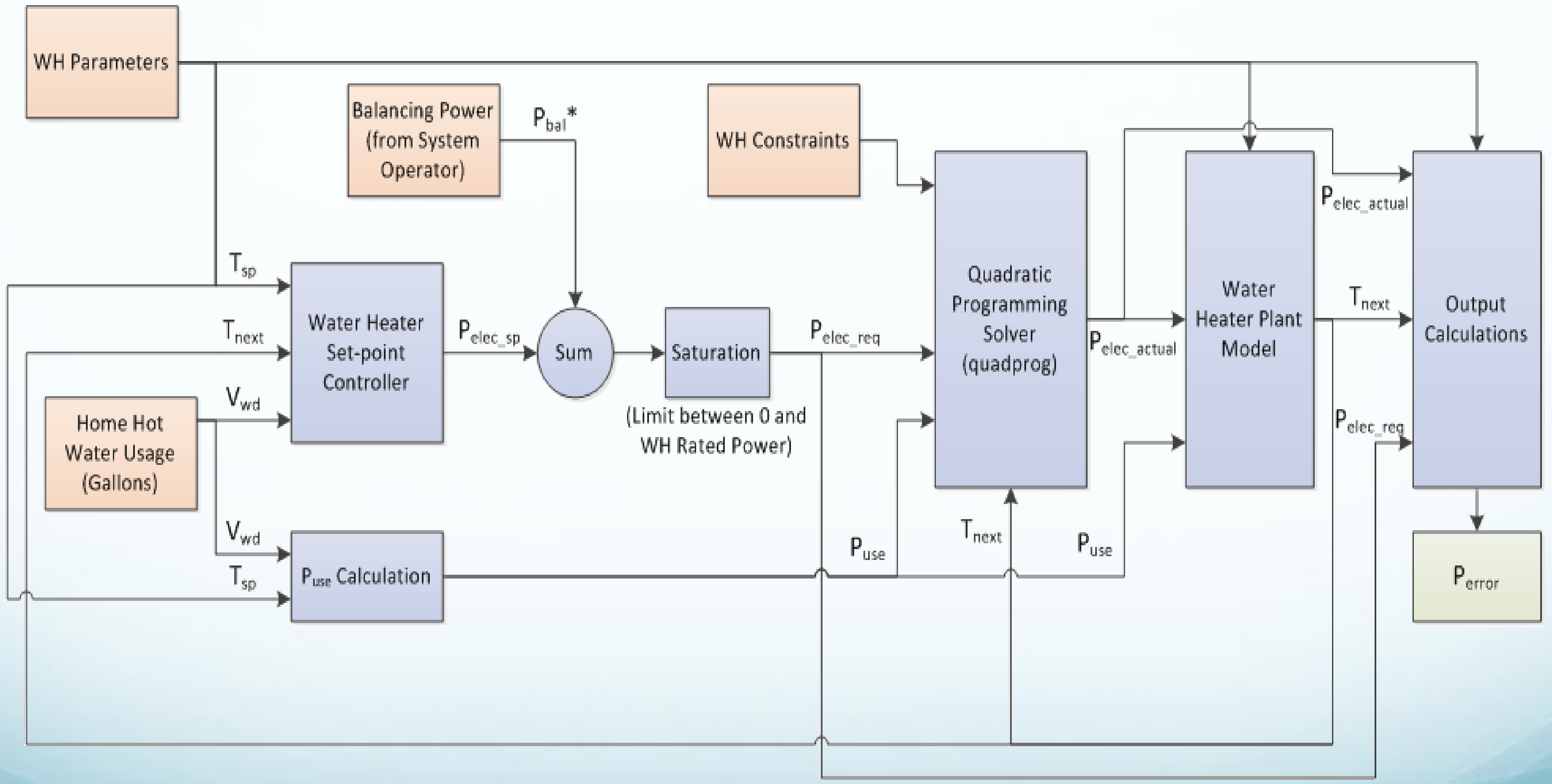


Water Heater Model Formulation

$$\underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}_{k+1}}_{\mathbf{x}(k+1)} = \underbrace{\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}_k}_{\mathbf{x}(k)} + \underbrace{\begin{bmatrix} B_{elec_1} & 0 \\ 0 & B_{elec_2} \end{bmatrix}}_{\mathbf{B}_u} \underbrace{\begin{bmatrix} P_{elec_1} \\ P_{elec_2} \end{bmatrix}_k}_{\mathbf{u}(k)} + \underbrace{\begin{bmatrix} 0 & B_{use_1} & B_{amb_1} & 0 & 0 \\ 0 & 0 & 0 & B_{use_2} & B_{amb_2} \end{bmatrix}}_{\mathbf{B}_v} \underbrace{\begin{bmatrix} P_{elec_{req}} \\ P_{use_1} \\ T_{amb_1} \\ P_{use_2} \\ T_{amb_2} \end{bmatrix}_k}_{\mathbf{v}(k)}$$

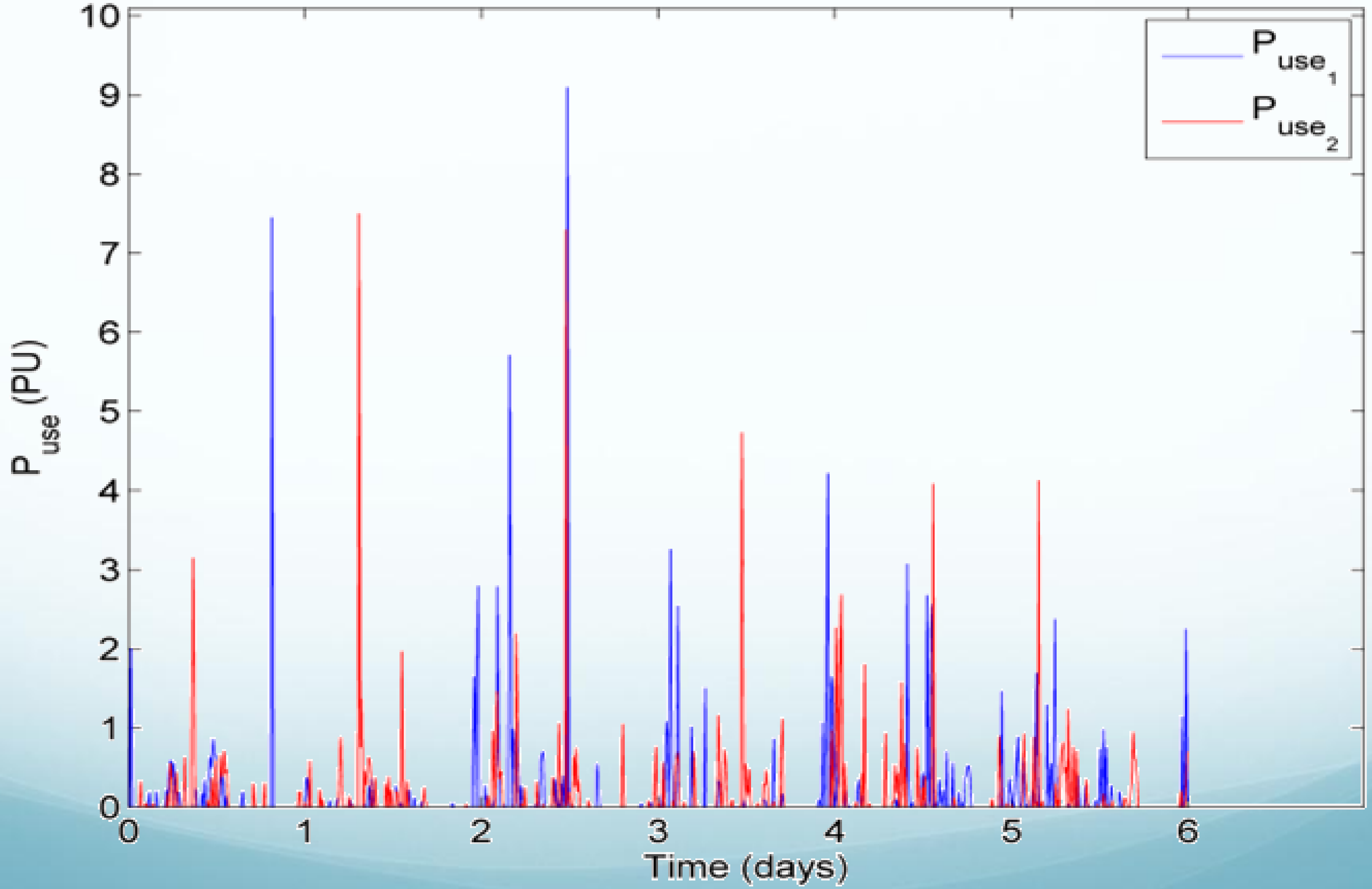
$$\underbrace{\begin{bmatrix} P_{error} \\ T_1 \\ T_{next_1} \\ T_2 \\ T_{next_2} \end{bmatrix}_k}_{\mathbf{y}(k)} = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ A_1 & 0 \\ 0 & 1 \\ 0 & A_2 \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}_k}_{\mathbf{x}(k)} + \underbrace{\begin{bmatrix} -1 & -1 \\ 0 & 0 \\ B_{elec_1} & 0 \\ 0 & 0 \\ 0 & B_{elec_2} \end{bmatrix}}_{\mathbf{D}_u} \underbrace{\begin{bmatrix} P_{elec_1} \\ P_{elec_2} \end{bmatrix}_k}_{\mathbf{u}(k)} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & B_{use_1} & B_{amb_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{use_2} & B_{amb_2} \end{bmatrix}}_{\mathbf{D}_v} \underbrace{\begin{bmatrix} P_{elec_{req}} \\ P_{use_1} \\ T_{amb_1} \\ P_{use_2} \\ T_{amb_2} \end{bmatrix}_k}_{\mathbf{v}(k)}$$

Schematic for MPC

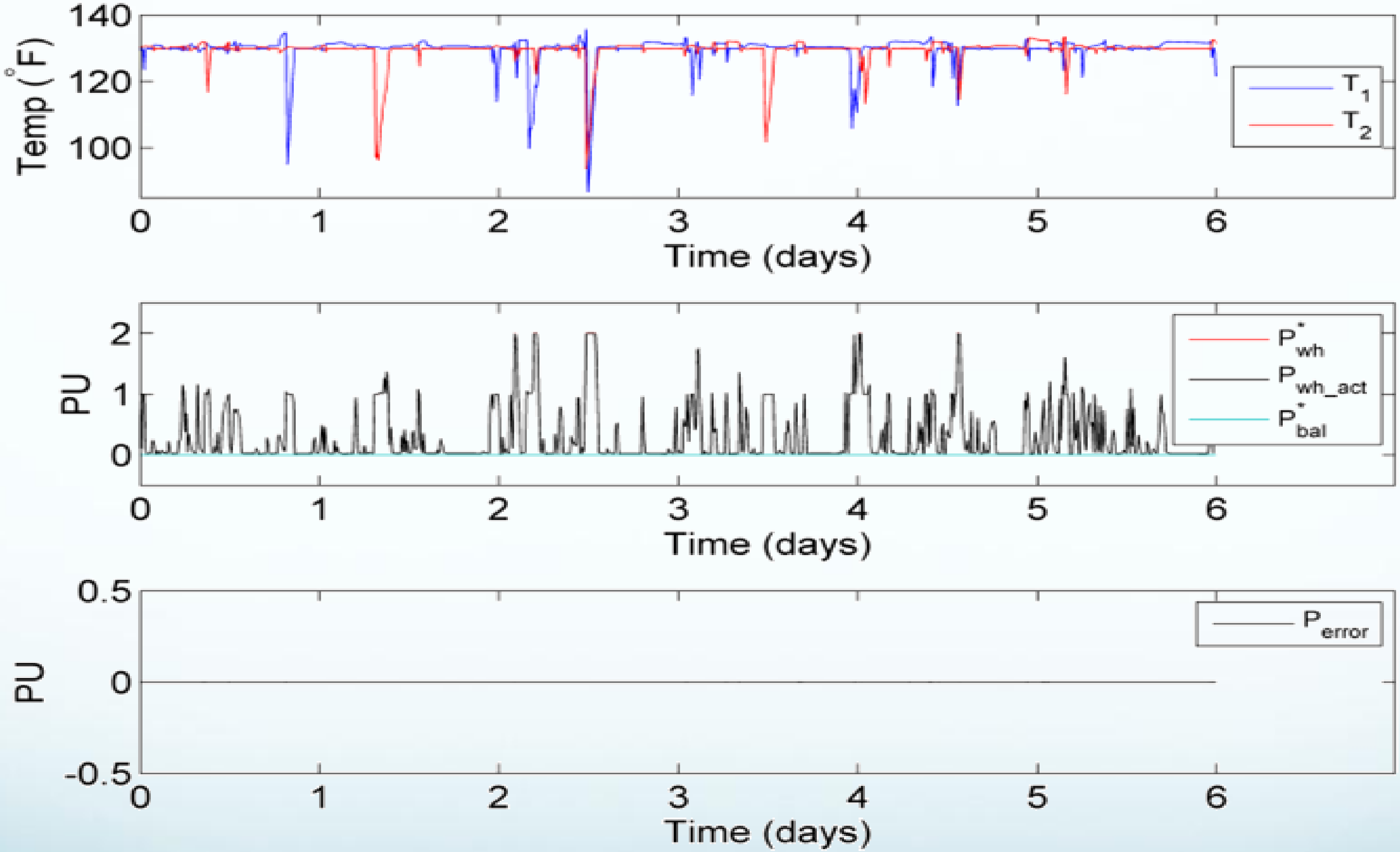


Results

Water Usage Data for 6 Days

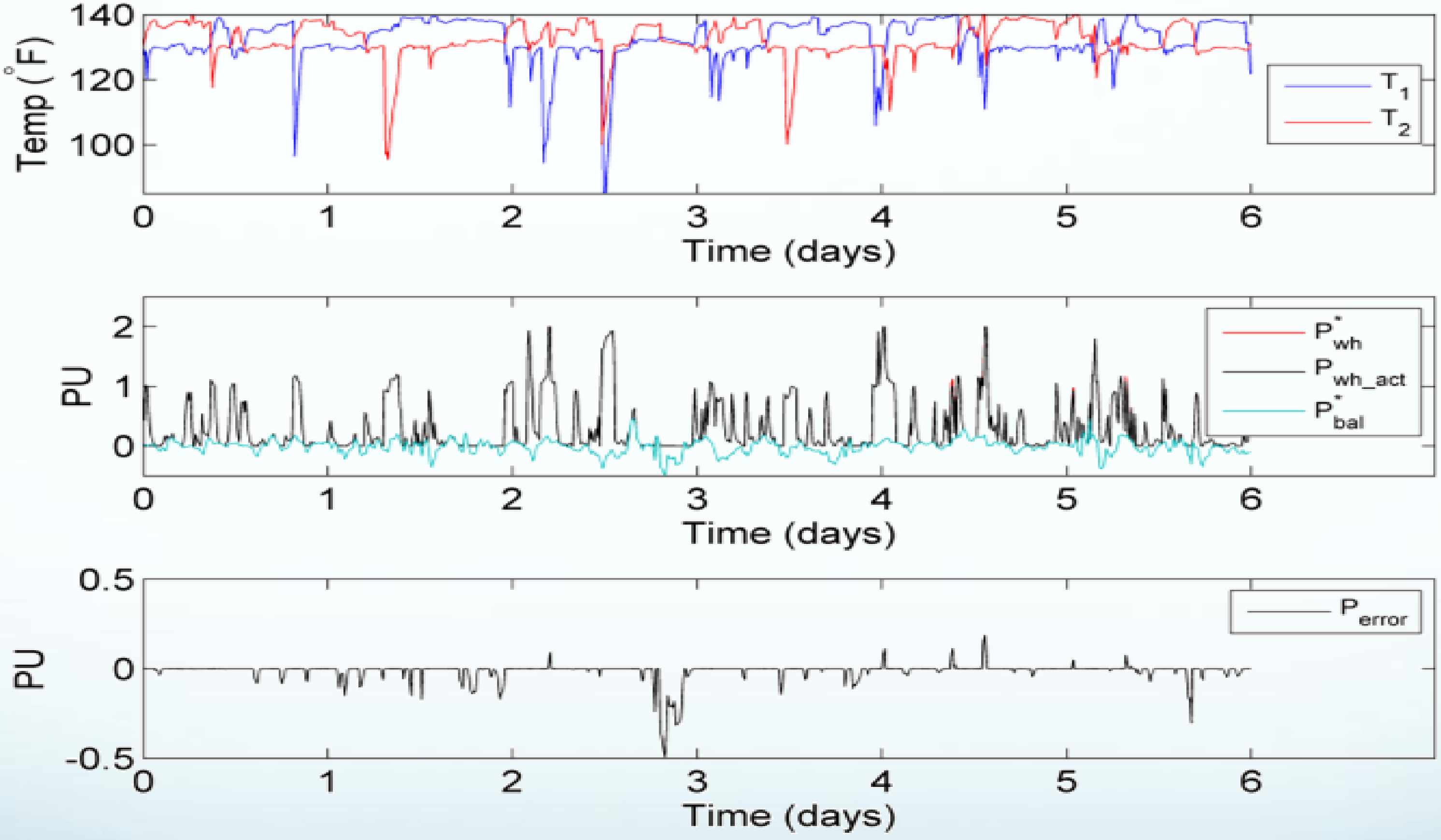


Zero Balancing Power

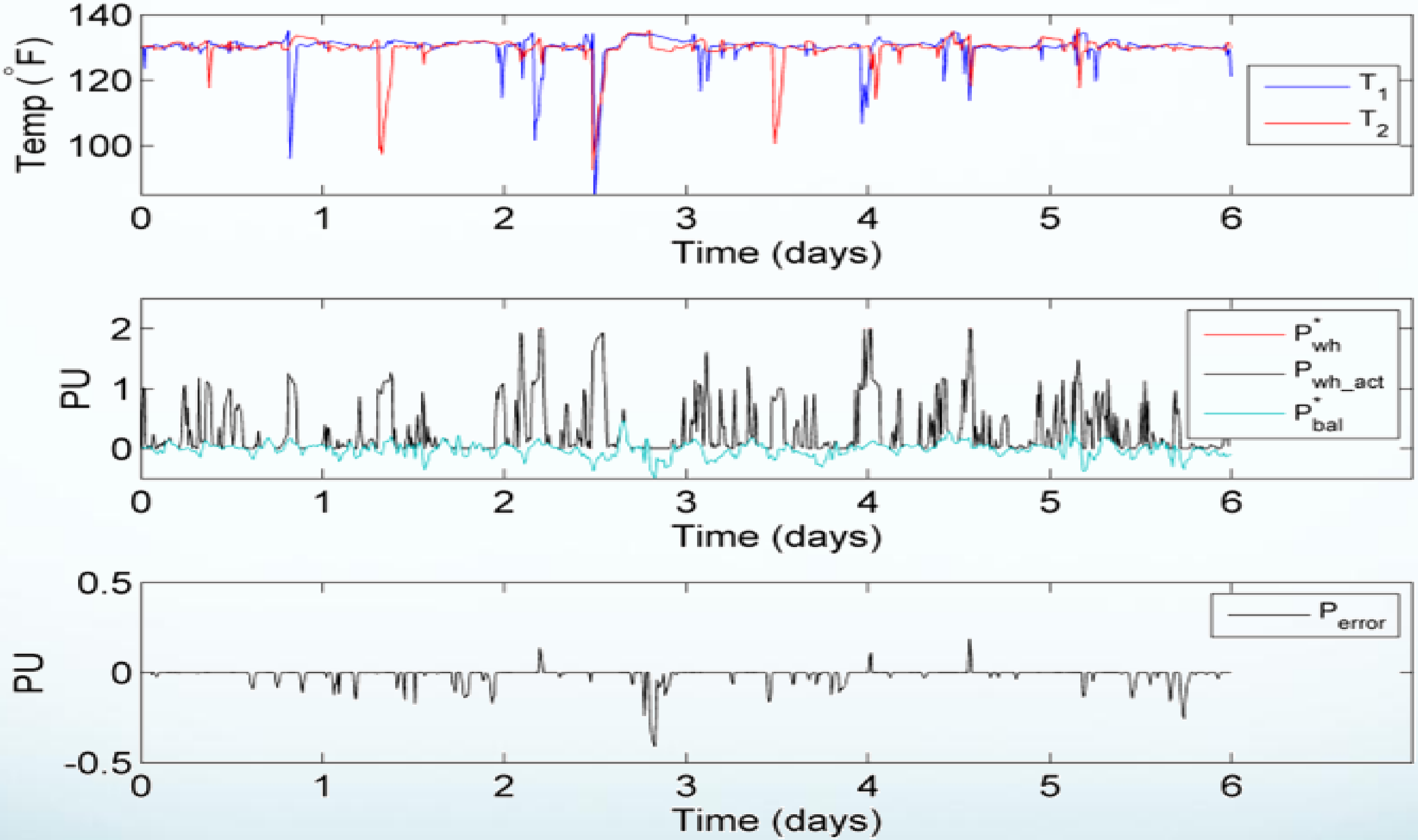


- No error when no balancing power is requested

60-min Scheduling with Prediction Off

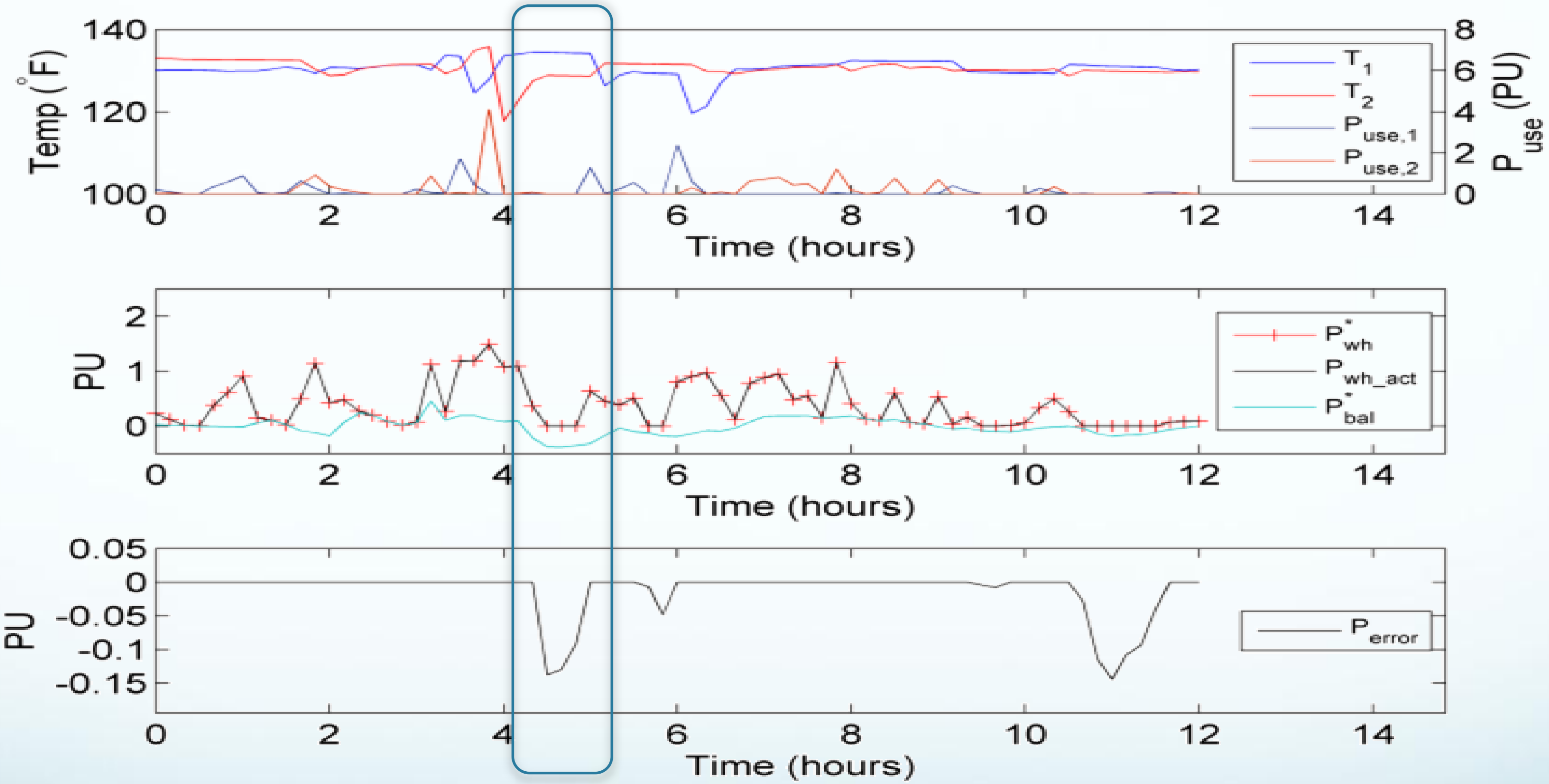


60-min Scheduling with Prediction On



- Error is reduced, and temperatures remain closer to set point

60-min Scheduling with Prediction On (12 hours)



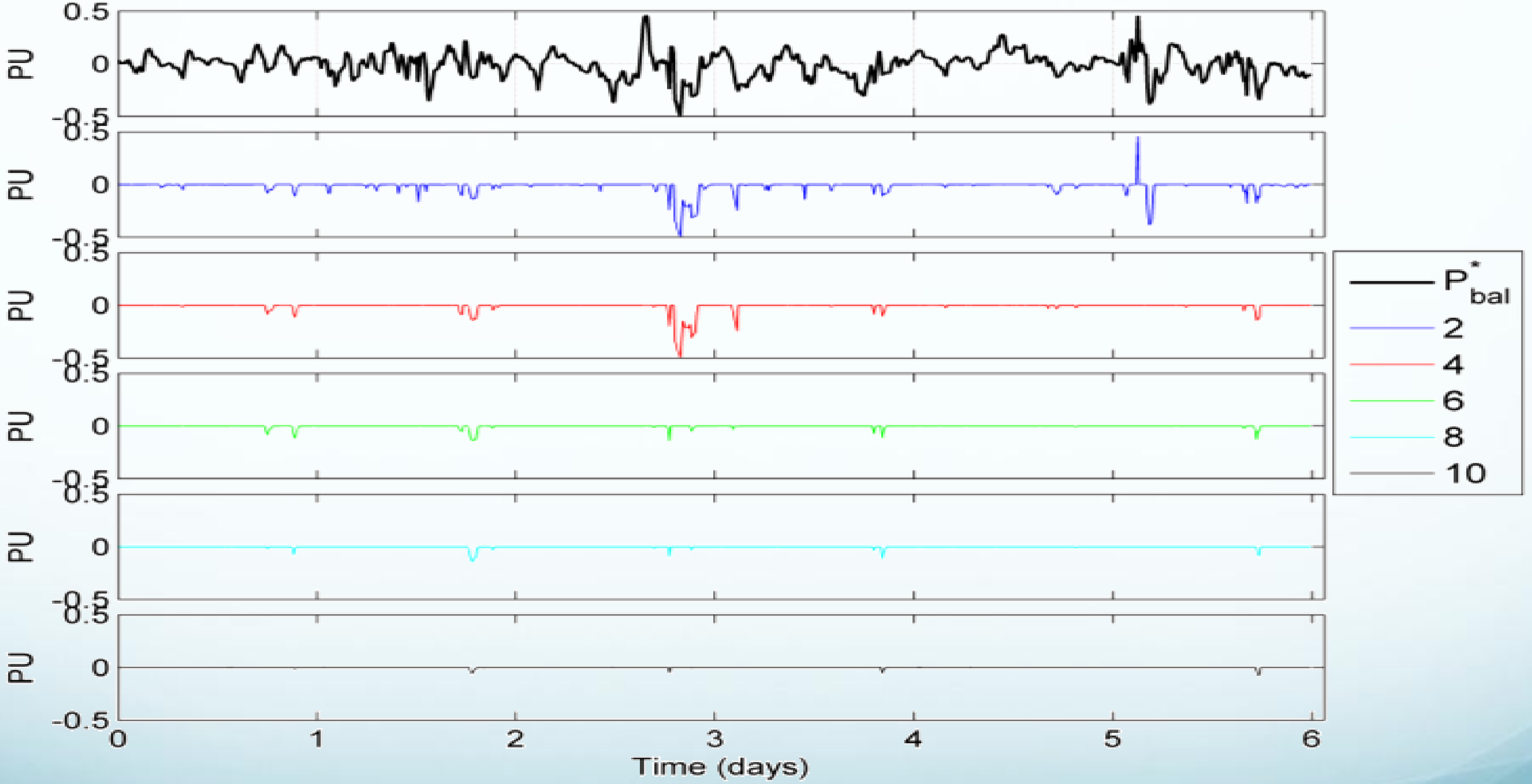
- Error occurs when minimal power usage and balancing power requested is negative

Summary of Results

Scheduling Interval	Zero P_{bal}	60 min.	60 min.	30 min.	30 min.
Predictive Control	On	Off	On	Off	On
P_{error} MAE (PU)	0.0000	0.0165	0.0148	0.0135	0.0122
P_{error} RMSE (PU)	0.0001	0.0552	0.0465	0.0455	0.0376
JQ (error) ("$")	0.0000	1.3021	0.9218	0.8829	0.6018
JQ (temp.) ("$")	0.0019	0.0045	0.0021	0.0044	0.0020
JQ (total) ("$")	0.0019	1.3067	0.9240	0.8873	0.6038

- Less error when predictions are used

Multiple Water Heaters



- Error is reduced as more water heaters are used

Summary of Results with Multiple Water Heaters

Number of water heaters	2	4	6	8	10
P_{error} MAE	0.0167	0.0098	0.0027	0.0016	0.0006
P_{error} RMSE	0.0601	0.0476	0.0152	0.0119	0.0053
JQ (error) (“$”)	1.5415	0.9652	0.0989	0.0606	0.0121
JQ (temp.) (“$”)	0.0029	0.0041	0.0066	0.0091	0.0130
JQ (total) (“$”)	1.5443	0.9693	0.1055	0.0697	0.0251

- Error is reduced as more water heaters are used

Conclusions

- Demonstrate MPC is feasible to control multiple water heaters
- Utilizing predictions improves the error and keeps temperatures close to set-point
- Error primarily occurs when negative balancing power is requested and water usage is low
- More water heaters with diversified water usage profiles drastically reduces error

Future Work

- Methodology to predict water usage
- Investigate implications of large-scale deployment of a demand response program
  - Multiple levels of control
- Explore impact of geographic separation between generation source and reserve resource

Questions?